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COMMENTS ON "VIBRATION ANALYSIS OF THIN CYLINDRICAL SHELLS USING THE WAVE PROPAGATION APPROACH"

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This paper [1] discussed an interesting method which employs the wave propagation approach to estimate the natural frequencies of closed circular cylindrical shells for various boundary conditions. Based on Love's motion equations, this method combines an exact frequency–wavenumber characteristics formula with appropriate beam functions in the axial direction to give more accurate predictions of natural frequencies of circular cylindrical shells. However, the advantage of this method over other major methods developed in the past was not fully discussed in this paper. A more detailed analysis of the method using the wave approach to solve Love's equations with regard to its validity and accuracy for simply supported, free–free and clamped–clamped boundary conditions was published by Wang and Lai [2, 3]. The purpose of this note is to highlight the conditions under which solutions obtained by solving Love's equations using the wave approach and beam functions will be accurate for thin circular cylindrical shells. In particular, we would like to discuss the condition under which the assumptions made in Love's equation are valid and the condition under which errors introduced by the use of beam functions become significant.

It is well known that Love's equations are for thin shells of which the thickness h is much less than its radius a and in which the shear deflection is small, so that the influence of rotatory inertia could be neglected. According to Soedel [4], the shear deformation should not be neglected for cases where the thickness dimension approaches a quarter wavelength of a modal bending wave. Therefore, the accuracy of the solutions obtained by solving Love's motion equations using the wave approach depends upon whether the assumptions made for Love's motion equations are valid. Figures 1 and 2 compare the frequency–wavenumber relationship obtained from Love's motion equations using the wave approach with that obtained by the finite-element method (FEM) [2] for $a/h = 20$ and $a/h = 5$ respectively. Here, the results are expressed in terms of non-dimensional parameters Ω and $k_z a$, where k_z is the wavenumber in the axial direction. The non-dimensional frequency parameter Ω is obtained by normalizing the frequency ω with the ring angular frequency, $\omega_r = (1/a)\sqrt{E/\rho}$, where E is Young's modulus and ρ is the density of the material. The FEM results were obtained from calculations made for cylindrical shell models with two radius/thickness ratios ($a/h = 5, 20$) and three different lengths ($l = 0{\cdot}2, 0{\cdot}5$ and 1 m). Only the results for simply supported shells are shown here

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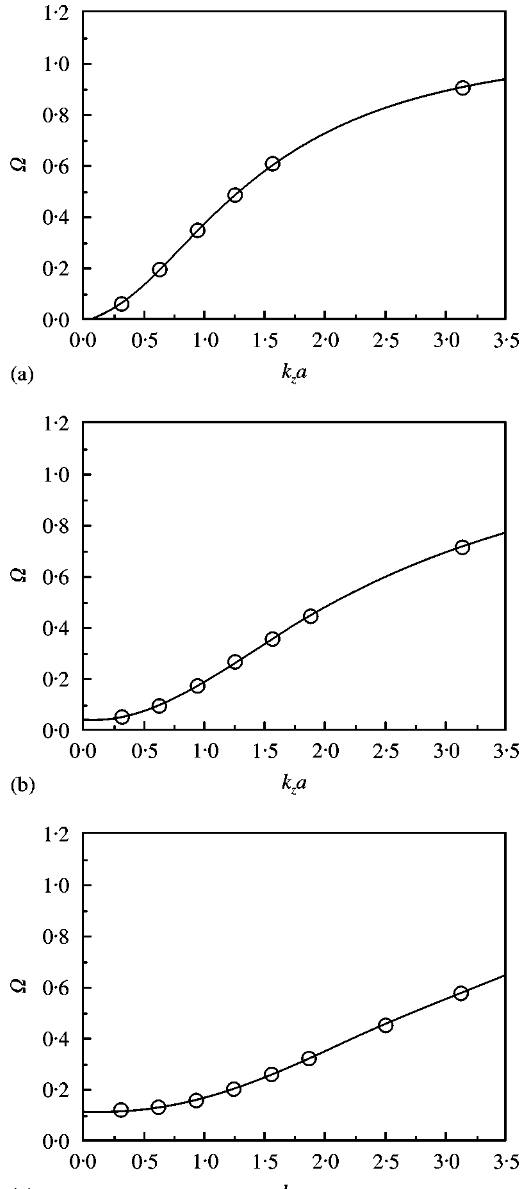

Figure 1. Comparison of the frequency–wavenumber relationship between the wave approach and FEM for $a/h = 20$: ——, wave approach; ○, FEM. (a) $n = 1$, (b) $n = 2$, (c) $n = 3$.

because the frequency–wavenumber relationship obtained from Love's equations with the wave approach using the beam functions is exact [2]. It can be seen from Figure 1 that for circular cylindrical shells with $a/h = 20$, the agreement between the results obtained by the wave approach and FEM is excellent within the range $k_z a < 3{\cdot}5$ for the circumferential mode order $n = 1$, 2 and 3. This is not surprising because the thin shell assumption is satisfied with $kh < 0{\cdot}23$. Here, $k = \sqrt{k_z^2 + k_\theta^2}$, $k_\theta = n/a$ is the wavenumber in the circumferential direction, and $kh = 0{\cdot}23$ corresponds to $k_z a = 3{\cdot}5$ and $n = 3$. On the other

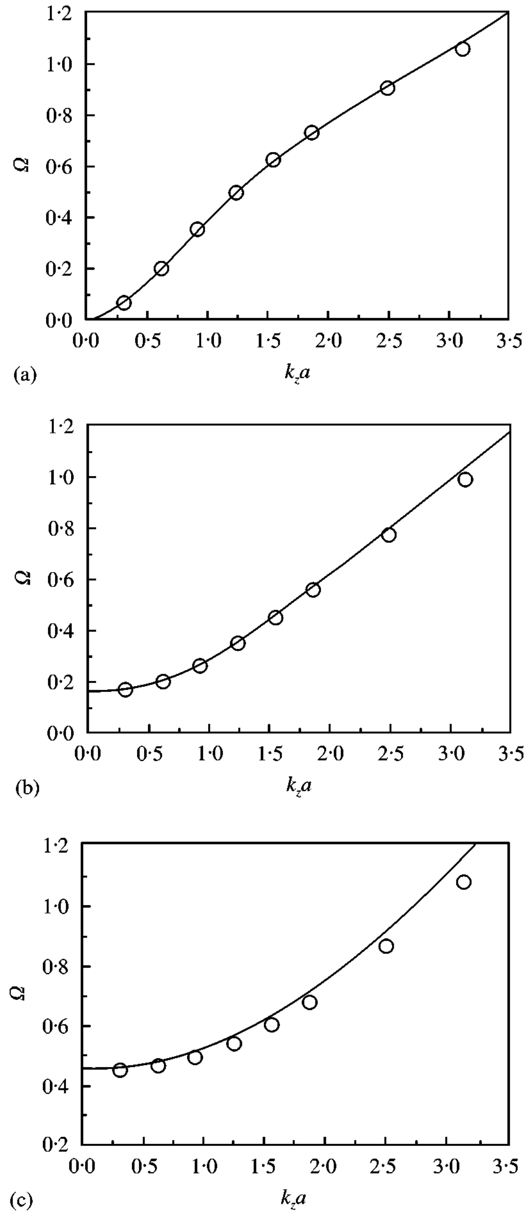

Figure 2. Comparison of the frequency–wavenumber relationship between the wave approach and FEM for $a/h = 5$: ——, wave approach; ○, FEM. (a) $n = 1$, (b) $n = 2$, (c) $n = 3$.

hand, for shells with $a/h = 5$, however, FEM results are slightly lower than the wave approach results within the range $k_z a < 3{\cdot}5$, especially as the circumferential mode order n increases (Figure 2). This is because for small a/h and higher order modes, the shear deflection and the rotatory inertia of the shell, which are not included in Love's equations, reduce the natural frequencies. Figure 2(c) indicates that for $k_z a = 3{\cdot}5$, $n = 3$ (corresponding to $kh = 0{\cdot}92$), the difference between the results obtained by Love's equations and FEM is

< 7%. Thus, the limited results presented in Figures 1 and 2 appear to suggest that the condition $kh < 1$ may generally guarantee that the frequency–wavenumber relationship obtained from Love's motion equations is applicable with an error of < 10%, with more accurate results at small kh.

When using the beam functions, errors may be introduced in the results due to neglecting the coupling of the vibration between the axial and the circumferential direction. The effects of such coupling are generally less important for long thin shells and for higher order modes. Figure 3 compares the natural frequencies of "short" (0·2 m long) circular cylindrical shells with both ends clamped calculated by the wave approach using beam functions and the FEM [2]. It can be seen that discrepancy is substantial for the shell with $a/h = 5$. Even for the shell with $a/h = 20$, the error is still large for lower order modes. So generally speaking, unless both ends of the shell are simply supported, the wave approach using beam functions is only good for relatively long cylindrical shells.

For very thin, long shells ($a/h > 300$), Soedel [5] obtained the formula (1) by solving Donnell-Mushtari-Vlasov motion equations for simply-supported boundary conditions. Donnell-Mushtari-Vlasov motion equations were based on Love's equations by introducing zero Gaussian curvature as an additional simplification. Equation (1) is much simpler and more straightforward to use than those of the wave approach [2]. Soedel [5] suggested that this equation can be used together with appropriate beam functions for the

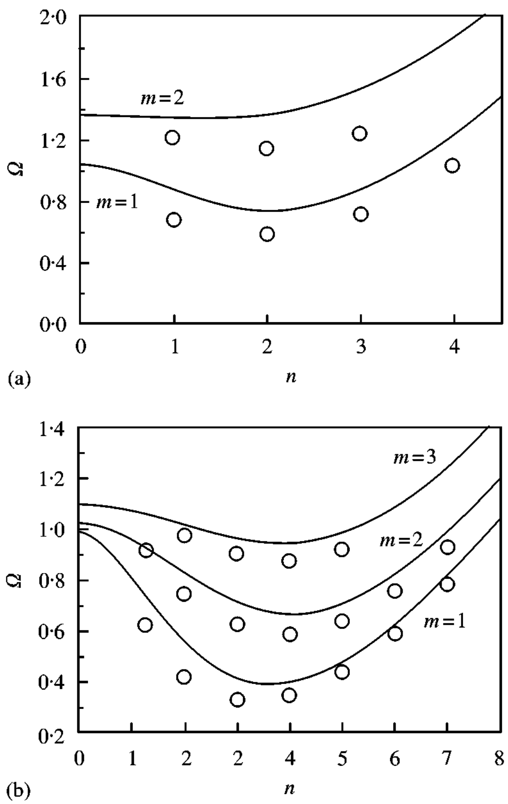

Figure 3. Comparison of the natural frequencies of a clamped–clamped circular cylindrical shells obtained by the wave approach and FEM: ——, wave approach; ○, FEM. (a) $a/h = 5$, $l = 0{\cdot}2$ m, (b) $a/h = 20$, $l = 0{\cdot}2$ m.

estimation of natural frequencies of closed circular cylindrical shells with different boundary conditions. This conjecture is indeed supported by the detailed analysis of the frequency-wavenumber relationship using the wave approach [2]

$$\omega^2 = \frac{D}{\rho h}[k_z^2 + k_\theta^2]^2 + \frac{K(1-\mu^2)}{\rho h a^2}\frac{k_z^4}{[k_z^2 + k_\theta^2]^2}. \tag{1}$$

It should be mentioned that references [2, 4] both suggested that the wave approach might only be applicable for the flexural vibration of cylindrical shells. For the other two in-plane vibrations, it is not clear yet whether the approach is effective or not.

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